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A NOTE ON WEAK STABILIZABILITY OF CONTRACTION SEMIGROUPS, (U)

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A NOTE ON WEAK STABILIZABILITY

OF

CONTRACTION SEMIGROUPS

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Abstract

A recent result on weak stabilizability is that the system $\dot{x} = Ax + Bu$, where A is the infinitesimal generator of a contraction semigroup over a Hilbert Space H , and B is linear bounded is weakly stabilizable if: (i) A has a compact resolvent and (ii) (A,B) is (approximately) controllable. In this note, we show that condition (i) is superfluous and (ii) can be weakened to (iii) the weakly unstable states are (approximately) controllable, which actually turns out to be a necessary condition. Indeed, if (i) is verified, (iii) is necessary and sufficient for strong stabilizability. Moreover, we give a simple, direct proof, using semigroup theoretic techniques, in particular obviating the need to invoke the "LaSalle Invariance Principle". The main tool is a decomposition applicable to all contraction semigroups which is derived from results of Sz. Nagy, C. Foias and S. R. Foguel.

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1. Introduction:

A standard result (see Wonham [9]) in finite dimension, is that the time invariant system

$$\dot{x} = Ax + Bu \quad (1.1)$$

is stabilizable by feedback

$$u = Kx$$

if and only if the unstable modes of the system are controllable. The extension of this result to infinite dimensional systems, i.e., where A is the infinitesimal generator of a C_0 semigroup, and B is linear bounded has been the subject of many investigations recently ([6], [8], [10],[11]). As may be expected, there are many non-equivalent notions of stability, depending on the topology used. The notion of "weak stability" would appear to be the weakest. Thus

Definition 1.1

A C_0 semigroup $T(t)$ over a Hilbert space H is weakly stable if $\forall x, y \in H, (T(t)x, y) \rightarrow 0$ as $t \rightarrow +\infty$.

Slemrod [6, Theorem 3.5] shows that if A generates C_0 contraction semigroup $T(t)$ over a Hilbert space H and B is a linear bounded transformation mapping a Hilbert space H_1 into H , the semigroup generated by $A - BB^*$ is weakly stable provided

- (i) A has a compact resolvent (note that for some reason this condition is stated in terms of A^* in [6], although of course the two are equivalent)
- (ii) (A, B) is (approximately) controllable.

In his proof, he uses the "LaSalle invariance principle". We shall show

(Theorem 3.1) that the assumption (i) is superfluous (and in fact is sufficient to yield strong stability) and (ii) can be considerably weakened. Moreover, our techniques are simpler and more directly semigroup theoretic, relying on a fundamental decomposition of contraction semigroups, following Sz. Nagy-Foias. We also incidentally indicate the relevance of the Sz. Nagy-Foias theory [7] to the whole problem.

We begin with some results of interest on their own.

2. Canonical Decomposition for Contraction Semigroups

In this section, we state two decomposition theorems for C_0 contraction semigroups, and merge them into one corollary. First, we recall some definitions:

Definition 2.1: Let H be a Hilbert space, and V be a bounded operator in H . We say that a subspace K reduces V if and only if

$$VK \subseteq K \text{ and } V^*K \subseteq K \quad (2.1)$$

Definition 2.2: A bounded operator V in H is

(i) Unitary if

$$V^*V = VV^* = I$$

(ii) Completely non unitary (c.n.u.) if there exists no subspace other than $\{0\}$ reducing V to a unitary operator.

Remark: It follows from (2.1) that both K and K^\perp reduce V and V^* .

Theorem 2.1 [Nagy-Foias] Let $T(t)$ be a C_0 contraction semigroup in a Hilbert space H . Then H can be decomposed into an orthogonal sum $H = H_u \oplus H_{cnu}$ where H_u and H_{cnu} are reducing subspaces for $T(t)$, such that

- (i) The restriction $T_u(t) = T(t)|_{H_u}$ of $T(t)$ to H_u is a unitary group.
- (ii) The restriction $T_{cnu}(t) = T(t)|_{H_{cnu}}$ of $T(t)$ to H_{cnu} is a c.n.u. semigroup.
- (iii) This decomposition (where of course H_u or H_{cnu} can be trivial) is unique and H_u can be characterized by

$$H_u = \{x \in H; \quad t \geq 0 \quad ||T(t)x|| = ||T^*(t)x|| = ||x||\} \quad (2.2)$$

$$\text{Moreover } H_u = \bar{K}_u, \text{ where } K_u = \mathcal{D}(A) \cap H_u, \quad (2.3)$$

and A denotes the infinitesimal generator of $T(t)$.

Proof: For the sake of completeness we sketch a proof. For more details see Sz. Nagy and Foias [7, pp. 9-10 and 136].

If we denote $D_{T(t)} = I - T^*(t)T(t)$ and $D_{T^*(t)} = I - T(t)T^*(t)$, then

$||T(t)x|| = ||T^*(t)x|| = ||x||$ is equivalent to

$$(D_{T(t)}x, x) = (D_{T^*(t)}x, x) = 0 \quad (2.4)$$

Since $T(t)$ is a contraction, $D_{T(t)}$ and $D_{T^*(t)}$ are both self adjoint nonnegative definite. It follows that $(2.4) \Leftrightarrow x \in N(D_{T(t)}) \cap N(D_{T^*(t)})$ where $N(\cdot)$ stands for the Null-Space of an operator. Therefore $H_u =$

$\bigcap_{t>0} [N(D_{T(t)}) \cap N(D_{T^*(t)})]$, which shows that it is closed. Using (2.2), it is easy to see that H_u is left invariant under $T(s)$ and $T^*(s)$ for any s .

To show (2.3) (which is not specifically contained in [7]), we first note that since H_u is closed,

$$\bar{K}_u = \overline{\mathcal{D}(A) \cap H_u} \subseteq H_u \quad (2.5)$$

Then, for any x in H_u , $R(\lambda, A)x = \int_0^\infty e^{-\lambda s} T(s)x \, ds$ is also in H_u , as

easily proved by checking that

$$T(t)T^*(t)R(\lambda, A)x = T^*(t)T(t)R(\lambda, A)x = R(\lambda, A)x$$

But $R(\lambda, A)x \in \mathcal{D}(A)$. Therefore $\forall x \in H_u, \lambda R(\lambda, A)x \in K_u$. But we know that $\lambda R(\lambda, A)x \rightarrow x$ as $\operatorname{Re} \lambda \rightarrow +\infty$ [1, p. 169]. Hence $H_u \subseteq \bar{K}_u$, which finally implies $H_u = \bar{K}_u$, associated with (2.5).

The following decomposition theorem is due to Foguel [4]. Here, we give a simple proof, using mainly elementary properties of contraction semigroups; this in turn shows the relevance of the contraction assumption to the stability problem.

Theorem 2.2 [Foguel] Let $T(t)$ be a C_0 contraction semigroup in a Hilbert space H . Let $W = \{x \in H; T(t)x \rightarrow 0 \text{ (weakly) as } t \rightarrow +\infty\}$. Then

(i) W reduces $T(s)$ for any s .

(ii) On W^\perp , $T(s)$ is reduced to a unitary group, or equivalently $W^\perp \subseteq H_u$.

(iii) W coincides with the subspace $\{x \in H; T^*(t)x \rightarrow 0 \text{ (weakly) as } t \rightarrow \infty\}$

Proof

•First, note that W is a closed subspace of H .

•Proof of (i): Let $x \in W$. Then, for any $s \geq 0$,

$$T(t)T(s)x = T(s+t)x \rightarrow 0 \text{ (weakly) as } t \rightarrow +\infty, \text{ and hence } T(s)x \in W. \quad (2.6)$$

In order to prove that $T^*(s)x \in W$, we need an intermediate result proved below.

$T^*(t)$ being a contraction, we have

$$\forall x \in H, \forall t_2 \geq t_1 \quad \|T^*(t_2)x\|^2 = \|T^*(t_2 - t_1)T^*(t_1)x\|^2 \leq \|T^*(t_1)x\|^2.$$

Therefore, for any x , $\|T^*(t)x\|^2$ is a non increasing function of t , bounded from below by 0. Hence, it converges as $t \rightarrow +\infty$. Therefore, for any fixed s

$$Z(t) = \|T^*(t)x\|^2 - \|T^*(t+s)x\|^2 \rightarrow 0 \text{ as } t \rightarrow +\infty.$$

$$\text{But } Z(t) = (T^*(t)x, T^*(t)x) - (T(s)T^*(t)x, T^*(t)x)$$

$$= ([I - T(s)T^*(s)]T^*(t)x, T^*(t)x)$$

$$= \|[I - T(s)T^*(s)]^{1/2} T^*(t)x\|^2$$

$$\text{Hence } \forall x \in H, \forall s \geq 0 \quad [I - T(s)T^*(s)]T^*(t)x \rightarrow 0 \text{ as } t \rightarrow +\infty \quad (2.7)$$

Multiplying it to the left by $T^*(s)$, we have

$$T^*(s) [I - T(s)T^*(s)]T^*(t)x = [I - T^*(s)T(s)]T^*(t+s)x.$$

Therefore, it follows that

$$\forall x \in H, \forall s \geq 0 \quad [I - T^*(s)T(s)]T^*(t)x \rightarrow 0 \text{ as } t \rightarrow \infty \quad (2.8)$$

Next, we use the fact that if $Y(t)$ is a bounded linear operator such

that $Y(t)x \rightarrow 0$ for any x in H , then $Y^*(t)x \rightharpoonup 0$ (weakly), for any x in H .

Applying this to (2.7) and (2.8), we get $\forall x \in H, \forall s \geq 0$

$$\begin{cases} T(t) [I - T(s)T^*(s)]x \rightharpoonup 0 \text{ (weakly) as } t \rightarrow +\infty & (2.9) \\ T(t) [I - T^*(s)T(s)]x \rightharpoonup 0 \text{ (weakly) as } t \rightarrow +\infty & (2.10) \end{cases}$$

Now, we are ready to complete the proof of (i). For, if we take x in W , we have $T(t)x \rightharpoonup 0$ (weakly) as $t \rightarrow +\infty$, and subtracting it from (2.9), we get

$$\begin{aligned} -T(t)T(s)T^*(s)x &= -T(t+s)T^*(s)x \rightharpoonup 0 \text{ (weakly) as } t \rightarrow +\infty \\ \Rightarrow T(t)T^*(s)x &\rightharpoonup 0 \text{ (weakly) as } t \rightarrow +\infty \end{aligned} \quad (2.11)$$

Grouping (2.6) and (2.11), we have

$$x \in W \Rightarrow \begin{cases} T(s)x \in W \\ T^*(s)x \in W \end{cases}$$

Therefore W and W^\perp reduce $T(s)$, for any s .

•Proof of (ii):

(2.9) and (2.10) can be interpreted as:

$$\text{For any } s \quad \begin{cases} R(D_{T^*(s)}) = \text{Range } [I - T(s)T^*(s)] \subseteq W \\ R(D_{T(s)}) = \text{Range } [I - T^*(s)T(s)] \subseteq W \end{cases} \quad (2.12)$$

$D_{T^*(s)}$ and $D_{T(s)}$ being self adjoint, we have

$$\begin{cases} R(D_{T^*(s)})^\perp = N(D_{T^*(s)}) \\ R(D_{T(s)})^\perp = N(D_{T(s)}) \end{cases}$$

Therefore (2.12) is equivalent to

$$\begin{aligned} \forall s \geq 0 \quad W^\perp &\subseteq N(D_{T(s)}) \cap N(D_{T^*(s)}) \\ \text{or} \quad W^\perp &\subseteq \bigcap_{s \geq 0} [N(D_{T(s)}) \cap N(D_{T^*(s)})] = H_u \end{aligned}$$

which completes the proof of (ii).

• Proof of (iii)

Let $x \in W$. Any y in H can be uniquely decomposed as $y = y_W + y_W^\perp$ where $y_W \in W$ and $y_W^\perp \in W^\perp$.

$$\begin{aligned} \text{Then } (T^*(t)x, y) &= (T^*(t)x, y_W + y_W^\perp) \\ &= (x, T(t)y_W) + (x, T(t)y_W^\perp) \end{aligned}$$

Since W^\perp reduces $T(t)$, $T(t)y_W^\perp \in W^\perp$ and $(x, T(t)y_W^\perp) = 0$.

But since $y_W \in W$, $T(t)y_W \rightarrow 0$ weakly as $t \rightarrow +\infty$ and $(T^*(t)x, y) = (x, T(t)y_W) \rightarrow 0$ as $t \rightarrow +\infty$ and $T^*(t)x \rightarrow 0$ (weakly) as $t \rightarrow +\infty$.

Reversing the role of $T(t)$ and $T^*(t)$, we can show that $T^*(t)x \rightarrow 0$ (weakly) $\Rightarrow T(t)x \rightarrow 0$ (weakly), which completes the proof.

We can unite the two theorems into the following corollary.

Corollary 2.1. Let H be a Hilbert space, and $T(t)$ a C_0 contraction semigroup in H . Then H can be decomposed into three orthogonal subspaces H_{cnu} , W_u and W^\perp , all reducing $T(t)$ and $T^*(t)$, such that

$$\begin{cases} W_u \oplus W^\perp = H_u \\ W_u \oplus H_{\text{cnu}} = W \end{cases} \quad (\text{with the above notations}).$$

It follows that,

• On H_{cnu} , $T(t)$ is completely non unitary, and weakly stable

- On W_u , $T(t)$ is unitary and weakly stable
- On W^l , $T(t)$ is unitary, and $\forall x \in W^l$, $T(t)x \not\rightarrow 0$ and $T^*(t)x \not\rightarrow 0$ as $t \rightarrow +\infty$.

Proof: Follows immediately from the two theorems.

The above result motivates the following definition:

Definition 2.3: Let $T(t)$ be a C_0 contraction semigroup over a Hilbert space H . Then $W = \{x; T(t)x \rightarrow 0 \text{ (weakly) as } t \rightarrow +\infty\}$ is called the weakly stable subspace.

W^l is called the "weakly unstable subspace" and elements of W^l are called "weakly unstable states".

[Of course $T(t)x \not\rightarrow 0$ as $t \rightarrow +\infty$ does not imply that $x \in W^l$.]

3. Necessary and sufficient condition for weak stabilizability of C_0 contraction semigroups.

In order to prove the main theorem of this section, we need some preliminary results. First, we recall what is meant by "controllability".

Definition 3.1 Consider the system

$$\dot{x} = Ax + Bu \quad (3.1)$$

where A generates a C_0 semigroup $T(t)$ over a Hilbert space H and B is a linear bounded operator mapping another Hilbert space H_1 into H . The set C of x in H , for which given any $\epsilon > 0$, there exist a $t > 0$ and $u(\cdot)$ in $L_2[(0, t); H_1]$ such that

$$\|x - \int_0^t T(t - \sigma) Bu(\sigma) d\sigma\| < \epsilon \quad (3.2)$$

is called the set of (approximately) controllable states. If $C = H$, the system is approximately controllable. See [1].

Lemma 3.1: With the above notations, C is a closed subspace and can be characterized by

$$C = \bigcup_{t \geq 0} \overline{\text{Range } [T(t)B]} \quad (3.3)$$

It follows that

$$C^\perp = \bigcap_{t \geq 0} N[B^*T^*(t)]. \quad (3.4)$$

C (resp. C^\perp) is called the (approximately) controllable (resp. uncontrollable) subspace.

Proof: See [1, pp. 207-210]

Next, we state two perturbation results.

Lemma 3.2: Let A be the infinitesimal operator of a C_0 semigroup $T(t)$ in a Hilbert space H , and D be a bounded operator in H . Then $A + D$ generates a C_0 semigroup $S(t)$ in H . Furthermore,

- (i) If A and D are self adjoint, so is $S(t)$, for any $t \geq 0$.
- (ii) If A and D are dissipative, $S(t)$ is a contraction semigroup.
- (iii) If A has a compact resolvent, so does $A + D$.
- (iv) If $T(t)$ is compact, for any $t > 0$, so is $S(t)$.

Proof: See [1, pp. 220-225]

Lemma 3.3 Let K be any bounded operator mapping a Hilbert space H_1 into H . Let $S(t)$ denote the semigroup generated by $A + BK$. Then $\forall t \geq 0$, $B^*T^*(t)x = 0$ if and only if $\forall t \geq 0$, $B^*S^*(t)x = 0$. (The (approximately) controllable subspace of (A, B) coincides with the one of $(A + BK, B)$).

Proof: Follows immediately from the identities

$$S^*(t)x = T^*(t)x + \int_0^t T^*(t - \sigma)K^*B^*S^*(\sigma)x d\sigma \quad (3.5)$$

and

$$T^*(t)x = S^*(t)x - \int_0^t S^*(t-\sigma)K^*B^*T^*(\sigma)x d\sigma \quad (3.6)$$

Theorem 3.1: Let A be the infinitesimal generator of a C_0 contraction semigroup $T(t)$ in a Hilbert space H , and B a bounded operator mapping another Hilbert space H_1 into H . Then, the system $\dot{x} = Ax + Bu$ is weakly stabilizable if and only if the "weakly unstable states" of $T(t)$ are (approximately) controllable, and $K = -B^*$ is a stabilizing feedback gain.

Proof:

Let C be the controllable subspace of (A,B) , as defined above. Let W be the weakly stable subspace of $T(t)$, as defined in Section 2. Then the theorem can be expressed as

$$(A,B) \text{ is weakly stabilizable} \iff W^\perp \subseteq C \iff C^\perp \subseteq W.$$

(i) Necessity

Suppose there exists a bounded operator K such that $A + BK$ generates a weakly stable semigroup $S(t)$. Then, let $x \in C^\perp$. By definition of C^\perp , we have $\forall t \geq 0 \ B^*T^*(t)x = 0$. Therefore, from (3.6), we get

$$\forall y \in H \quad (T^*(t)x, y) = (S^*(t)x, y) = (x, S(t)y) \rightarrow 0 \text{ as } t \rightarrow +\infty, \text{ by assumption.}$$

Therefore $T^*(t)x \rightarrow 0$ (weakly) as $t \rightarrow +\infty$, and since $T^*(t)$ is a contraction, we can use (iii) of Theorem 2.2 to prove that $T(t)x \rightarrow 0$ (weakly) as $t \rightarrow +\infty \Rightarrow x \in W$. So $C^\perp \subseteq W$. Q.E.D.

(ii) Sufficiency.

Assume $C^\perp \subseteq W$. Let $K = -B^*$ be the feedback gain. Then $-BB^*$ is obviously a bounded dissipative operator, and by (ii) of Lemma 3.2, $A - BB^*$ generates a contraction semigroup $S(t)$. Then, applying the

Theorem 2.1 to $S(t)$, we obtain a decomposition of H into two orthogonal subspaces H_u^S , reducing $S(t)$ to a unitary group, and H_{cnu}^S , reducing $S(t)$ to a c.n.u. semigroup. Then, by Corollary 2.1, we have

$$\forall x \in H_{cnu}^S, S(t)x \rightarrow 0 \text{ (weakly) as } t \rightarrow +\infty$$

Therefore, it only remains to prove that $S(t)$ is weakly stable on H_u^S .

Define K_u^S as in Theorem 2.1. Then, for any x in $K_u^S \subseteq \mathcal{D}(A)$ we have

$$\forall t \geq 0 \frac{d}{dt} \|S^*(t)x\|^2 = ((A^* - BB^*)S^*(t)x, S^*(t)x) + (S^*(t)x, (A^* - BB^*)S^*(t)x) = 0$$

Since A^* and $-BB^*$ are dissipative, the above equation implies that

$$\forall t \geq 0 B^*S^*(t)x = 0.$$

But, by Lemma 3.3, this implies that $\forall t B^*T^*(t)x = 0$ or equivalently $x \in C^\perp$. So

$$x \in K_u^S \Rightarrow x \in C^\perp \quad (3.7)$$

But by assumption $C^\perp \subseteq W$. Therefore

$$x \in K_u^S \Rightarrow x \in W \quad (3.8)$$

Using (3.6) and (3.7), we get:

$$\forall t \geq 0, \forall x \in K_u^S \quad S^*(t)x = T^*(t)x.$$

Since $x \in W$, $T^*(t)x \rightarrow 0$ (weakly) as $t \rightarrow +\infty$, by (iii) of Theorem 2.2.

So does $S^*(t)x$, and so does $S(t)x$ by the same argument.

Therefore $\forall x \in K_u^S, S(t)x \rightarrow 0$ (weakly) as $t \rightarrow +\infty$. Since K_u^S is dense in H_u^S (Theorem 2.1), and $\|S(t)\| \leq 1$, then, for any x in H_u^S , $S(t)x \rightarrow 0$ (weakly) as $t \rightarrow +\infty$, by the triangular inequality. This completes the proof.

Corollary 3.1: If A generates a C_0 contraction semigroup and has a compact resolvent, the condition of Theorem 3.1 is necessary and sufficient for the strong stabilizability of (A,B) . In particular, $A-BB^*$ generates a strongly stable contraction semigroup.

Proof:

(i) Necessity: Follows from the fact that strong stability \Rightarrow weak stability.

(ii) Sufficiency: From (iii) of Lemma 3.2, $A - BB^*$ has a compact resolvent $R(\lambda, A - BB^*)$ and generates a contraction semigroup $S(t)$, which is weakly stable by Theorem 3.1. Let λ_0 be a point in the resolvent set of $A - BB^*$. Then, for any x in $\mathcal{D}(A)$, there exists a y in H such that $x = R(\lambda_0, A - BB^*)y$.

$$\begin{aligned} \text{Then } S(t)x &= S(t)R(\lambda_0, A - BB^*)y \\ &= R(\lambda_0, A - BB^*)S(t)y \end{aligned}$$

Since $\forall y \in H, S(t)y \rightarrow 0$ (weakly) as $t \rightarrow +\infty$, and since $R(\lambda_0, A - BB^*)$ is compact

$$\forall x \in \mathcal{D}(A) \quad S(t)x \rightarrow 0 \text{ as } t \rightarrow +\infty.$$

Since $\mathcal{D}(A)$ is dense in H , and $\|S(t)\| \leq 1$,

$$\forall x \in H, S(t)x \rightarrow 0 \text{ as } t \rightarrow +\infty \quad \text{Q.E.D.}$$

Corollary 3.2: If A generates a C_0 contraction self adjoint semigroup, the condition of Theorem 3.1 is necessary and sufficient for the strong stabilizability of (A,B) . In particular $A - BB^*$ generates a strongly stable self adjoint contraction semigroup.

Proof:

(i) As in Corollary (3.1)

(ii) Sufficiency

By (i) and (ii) of Lemma (3.2), $A - BB^*$ generates a self adjoint contraction semigroup $S(t)$ which is weakly stable, by Theorem 3.1.

Therefore $\forall x \in H (S(2t)x, x) \rightarrow 0$ as $t \rightarrow +\infty$,

But $(S(2t)x, x) = (S(t)S(t)x, x) = \|S(t)x\|^2 \rightarrow 0$ as $t \rightarrow +\infty$, Q.E.D.

Corollary 3.3: If A generates a compact contraction semigroup, the condition of Theorem 3.1 is necessary and sufficient for the exponential stabilizability of (A, B) . In particular, $A - BB^*$ generates an exponentially stable contraction semigroup.

Proof: Necessity as before.

Sufficiency follows from the fact that for a compact semigroup

Weak Stability \Rightarrow Exponential Stability. See [3].

This corollary is also a consequence of the sufficient condition proven in [8].

4. Conclusion and Remarks

Triggiani [8] has given a number of counterexamples of systems which are (approximately) controllable (A.C.) but not strongly stabilizable (S.S.). This paper shows that the A.C. of the weakly unstable states implies the weak stabilizability (W.S.) of the system, provided $T(t)$ is a contraction semigroup*. In particular, wave equations, which generate unitary groups in general, can be weakly stabilized if (A, B) is A.C. and strongly stabilized if in addition, the domain happens to be compact (thus insuring the compactness of the resolvent).

For further results involving semigroups other than contractions, we refer to [2].

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* The author was informed by one of the referees that a sufficient condition for weak stabilizability (A.C. \Rightarrow W.S.) was independently and simultaneously obtained by R. E. O'Brien [5]. Our result is a necessary and sufficient condition which shows that the system need not be A.C. on the whole space, in order to be weakly stabilized.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A recent result on weak stabilizability is that the system $\dot{x}_n = Ax + Bu$, where A is the infinitesimal generator of a contraction semigroup over a Hilbert Space H , and B is linear bounded is weakly stabilizable if: (i) A has a compact resolvent and (ii) (A, B) is (approximately) controllable. In this note, we show that condition (i) is superfluous and (ii) can be weakened to (iii) the weakly unstable states are (approximately) controllable, which actually turns out to be a necessary condition. Indeed, if (i) is verified, (iii) is necessary and sufficient for strong stabilizability. Moreover, we give a simple, direct proof, using \mathcal{H}_∞ norm (over)		

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→ semigroup theoretic techniques, in particular obviating the need to invoke the "LaSalle Invariance Principle". The main tool is a decomposition applicable to all contraction semigroups which is derived from results of Sz. Nagy, C. Foias and S. R. Foguel.

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